

Phys 231 Exam 4

April 20, 2018

NAME: Solutions

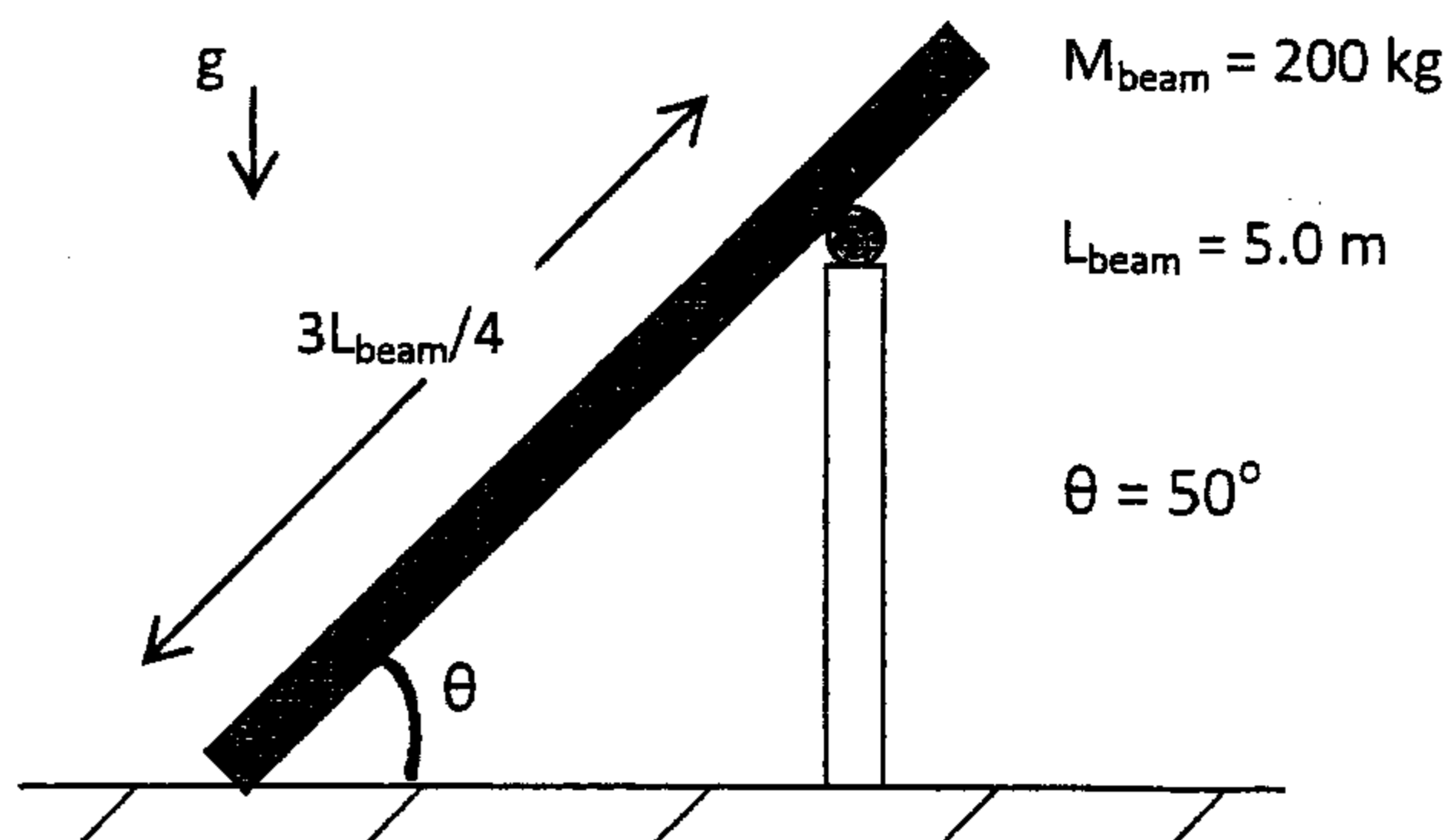
Student I. D. No. \_\_\_\_\_

# ALL UNITS!

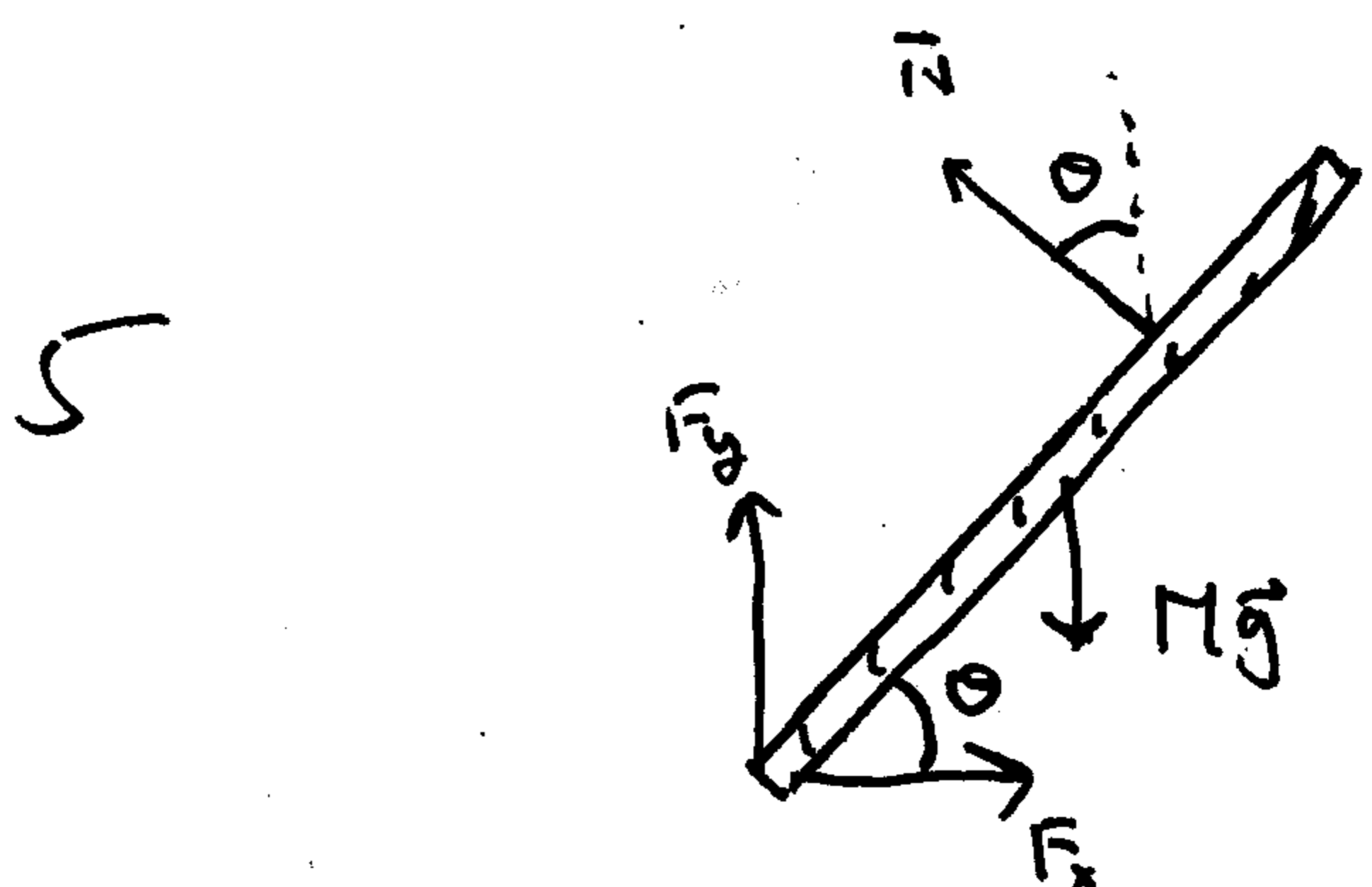
## All the Time

Remember that I must be able to follow your work. If I cannot figure out how you got the answer then no credit is given even if the answer is correct.

1) (15 pts) The diagram below depicts a uniform beam resting up against a frictionless roller. The other end is resting on the ground. The length of the beam is 5.0 m.



a) In the space below, draw a force diagram for the beam.



b) Choose the point of contact with the ground as your origin. What torque is exerted on the beam by the roller? Torque is a vector.

5

$$\tau_N - \frac{MgL}{2} \cos \theta = 0$$

$$\tau_N = \left( \frac{200 \text{ kg}}{2} \right) (9.8 \text{ m/s}^2) (5.0 \text{ m}) (\cos 50^\circ)$$

$$= 3150 \text{ Nm } \odot$$

c) Calculate the force on the beam at the point of contact with the ground. Use the i, j notation for vector.

$$N\left(\frac{3L}{4}\right) - \frac{Mg}{2}L \cos \theta = 0$$

$$N = \frac{2}{3} Mg \cos \theta = \left(\frac{2}{3}\right)(200 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2})(\cos 50^\circ)$$
$$= 840 \text{ N}$$

$$F_x = N \sin \theta = (840 \text{ N})(\sin 50^\circ) = 643 \text{ N}$$

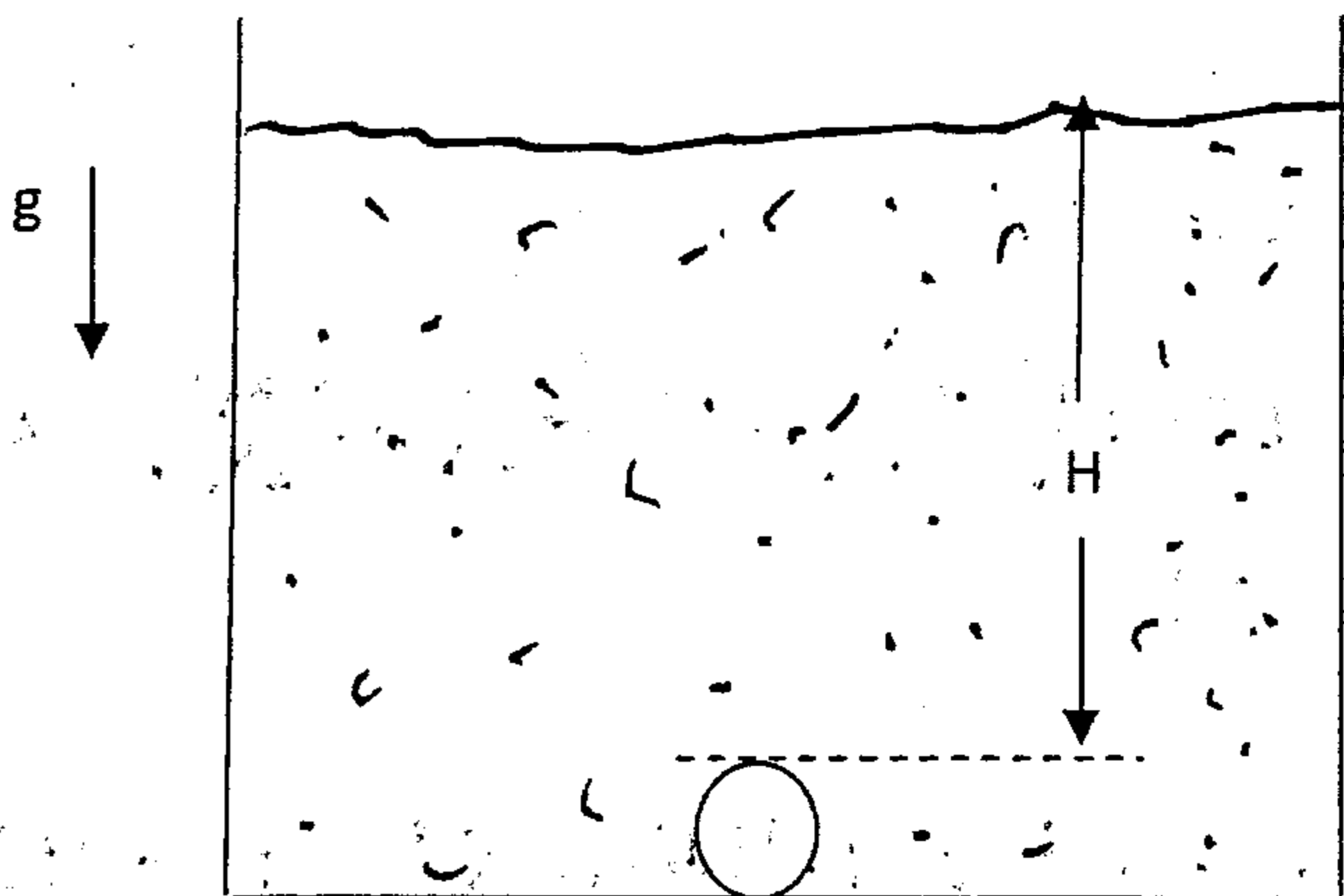
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$$F_y + N \cos \theta - Mg = 0$$

$$F_y = Mg - N \cos \theta$$
$$= (200 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2}) - (840 \text{ N})(\cos 50^\circ)$$
$$= 1420 \text{ N}$$

$$\vec{F} = (643 \hat{i} + 1420 \hat{j}) \text{ N}$$

2) (15 pts) The diagram below shows a sphere of radius  $R_s = 10$  cm and density  $= 3.0 \times 10^2$  kg/m<sup>3</sup> immersed in water (density  $= 1.0 \times 10^3$  kg/m<sup>3</sup>). The depth  $H$  is 1.5 m.



a) Calculate the buoyancy force acting on the sphere.

$$F_b = \rho_{H_2O} V g \quad V = \frac{4}{3} \pi R_s^3$$

$$F_b = (1.0 \times 10^3 \frac{\text{kg}}{\text{m}^3}) \left( \frac{4\pi}{3} \right) (0.10 \text{ m})^3 (9.8 \frac{\text{m}}{\text{s}^2})$$

$$= 41.1 \text{ N}$$

b) The force of gravity is also acting on the sphere. Calculate the net force (gravity plus buoyancy) acting on the sphere.

$$W = mg = \left( \frac{4}{3} \pi R_s^3 \right) \rho g$$

$$= \left( \frac{4}{3} \pi \right) (0.10 \text{ m})^3 (3.0 \times 10^2 \frac{\text{kg}}{\text{m}^3}) (9.8 \frac{\text{m}}{\text{s}^2})$$

$$= 12.3 \text{ N} \quad m = \frac{12.3 \text{ N}}{9.8 \frac{\text{m}}{\text{s}^2}} = 1.26 \text{ kg}$$

$$F_{\text{net}} = (41.1 - 12.3) \text{ N} = 28.8 \text{ N up}$$

c) Assume that there is no drag force from the water ("physics water") acting on the sphere. If the sphere released from rest, how fast is it going when it reaches the surface of the water?

$$v^2 = 2aH \quad a = \frac{F_{\text{net}}}{m}$$

$$\begin{aligned} v &= \sqrt{\frac{2F_{\text{net}}H}{m}} \\ &= \left[ \frac{(2)(28.8 \text{ N})(1.5 \text{ m})}{1.26 \text{ kg}} \right]^{1/2} \\ &= 8.28 \text{ m/s} \end{aligned}$$

3) (20 pts) The graph below shows the **velocity** versus time for a harmonic oscillator. The mass of the oscillator is 0.75 kg.

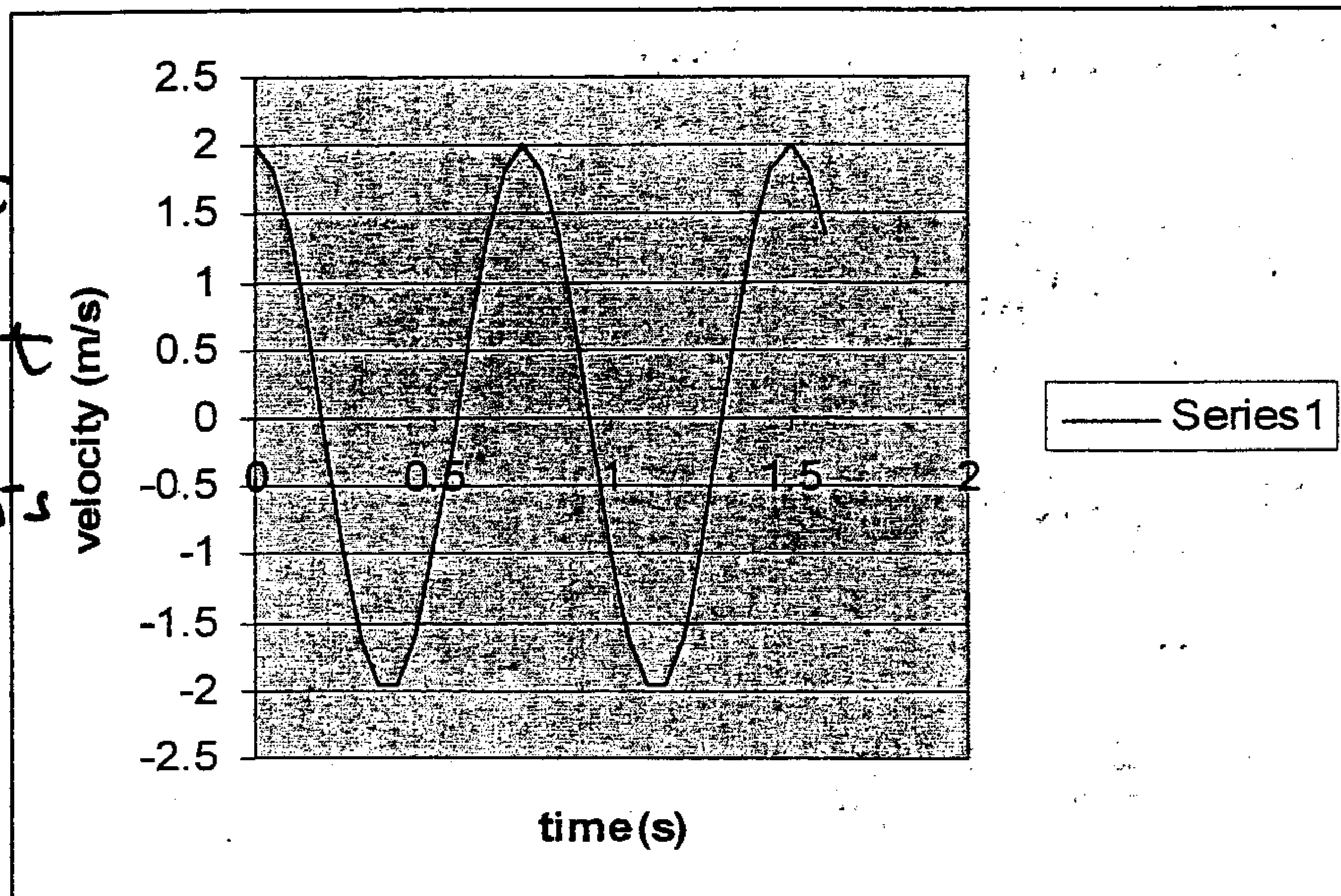
$$V(t) = V_m \cos \omega t$$

$$X(t) = \frac{V_m \sin \omega t}{\omega}$$

$$a(t) = -V_m \omega \sin \omega t$$

$$\omega = \frac{2\pi}{T} \quad T = 0.75 \text{ s}$$

$$\omega = 8.38 \text{ s}^{-1}$$

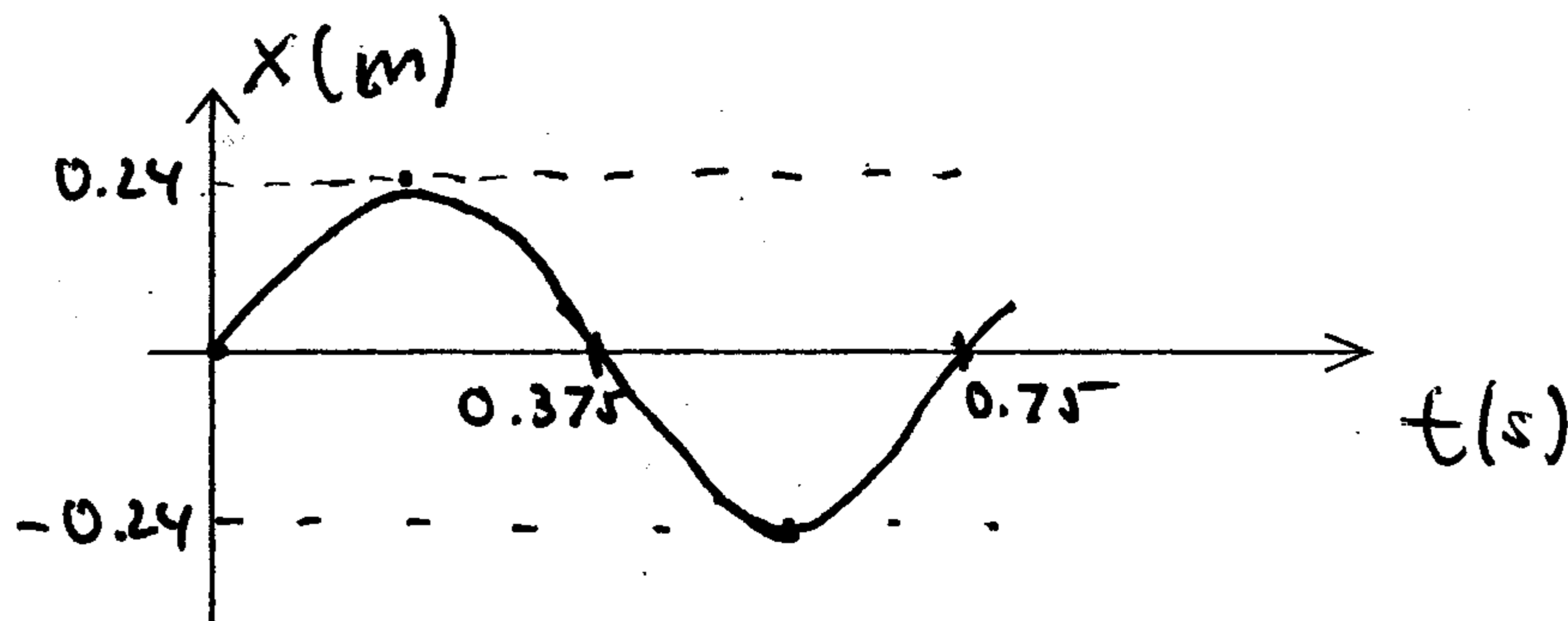


4 a) In the space below, sketch a graph of position (vertical axis) versus time (horizontal axis) for this oscillator. In all graphs, be certain to include axis labels, including units, and a numerical scale with hash marks.

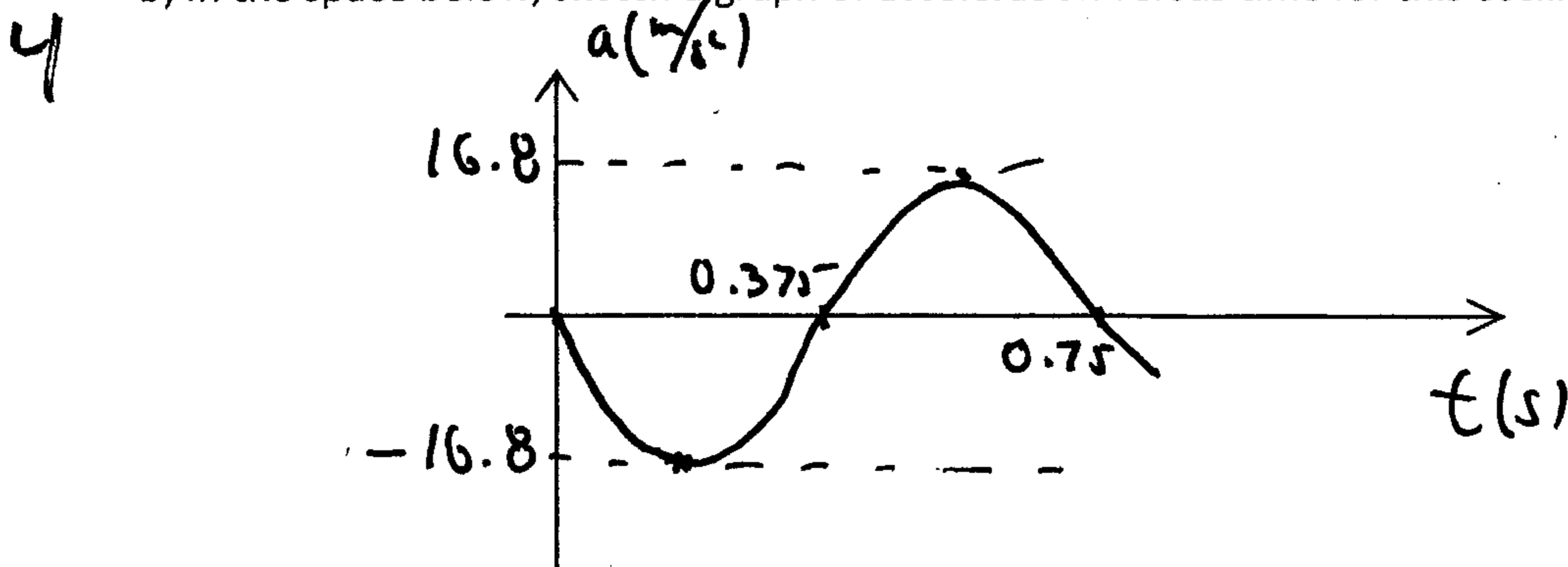
$$X_m = \frac{V_m}{\omega}$$

$$= \frac{2.0 \frac{\text{m}}{\text{s}}}{8.38 \text{ s}^{-1}}$$

$$= 0.24 \text{ m}$$



b) In the space below, sketch a graph of acceleration versus time for this oscillator.

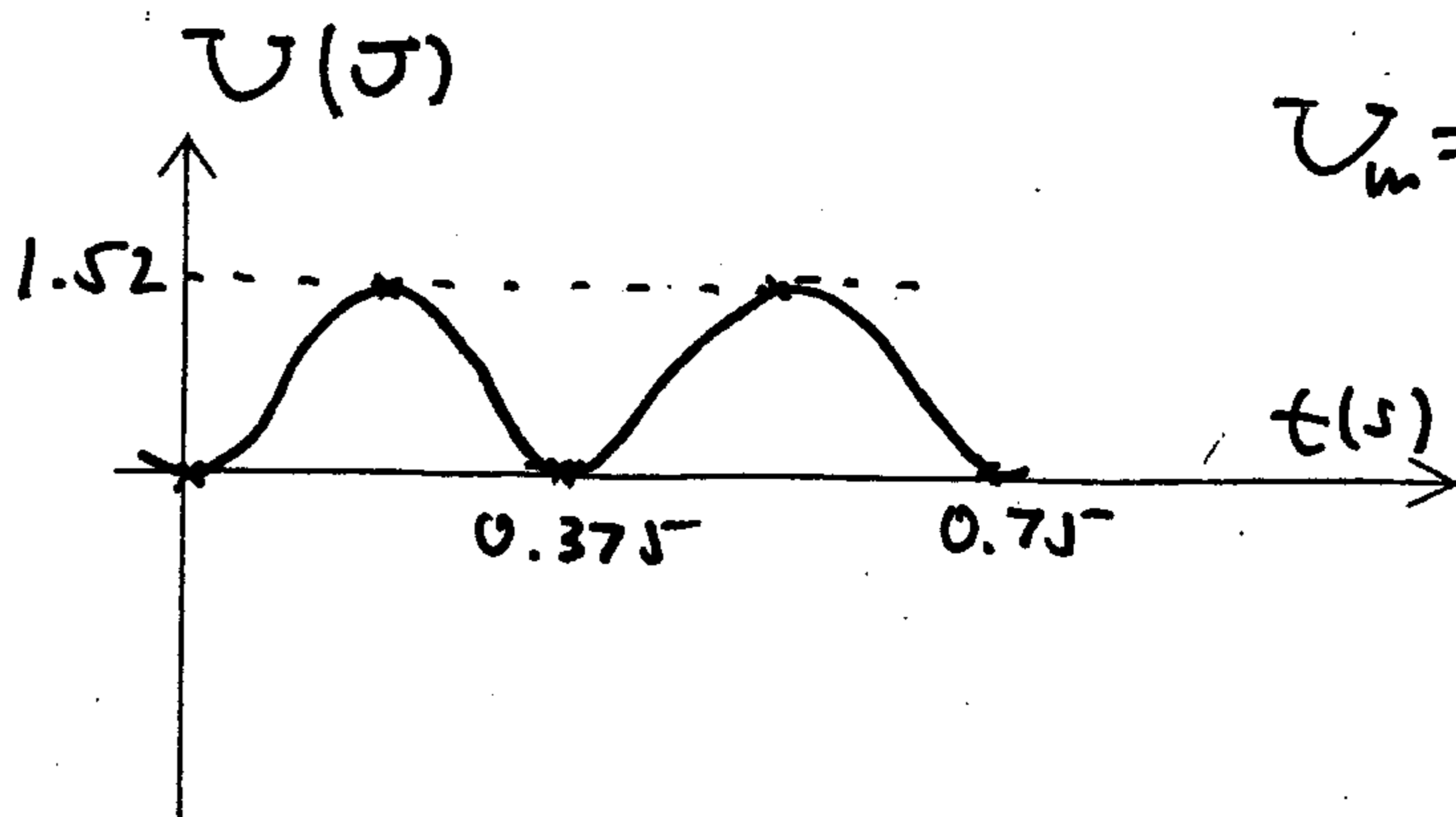


$$a_m = V_m \omega = (2.0 \frac{\text{m}}{\text{s}})(8.38 \text{ s}^{-1})$$

$$= 16.8 \frac{\text{m}}{\text{s}^2}$$

c) In the space below, sketch a graph of potential energy versus time.

4



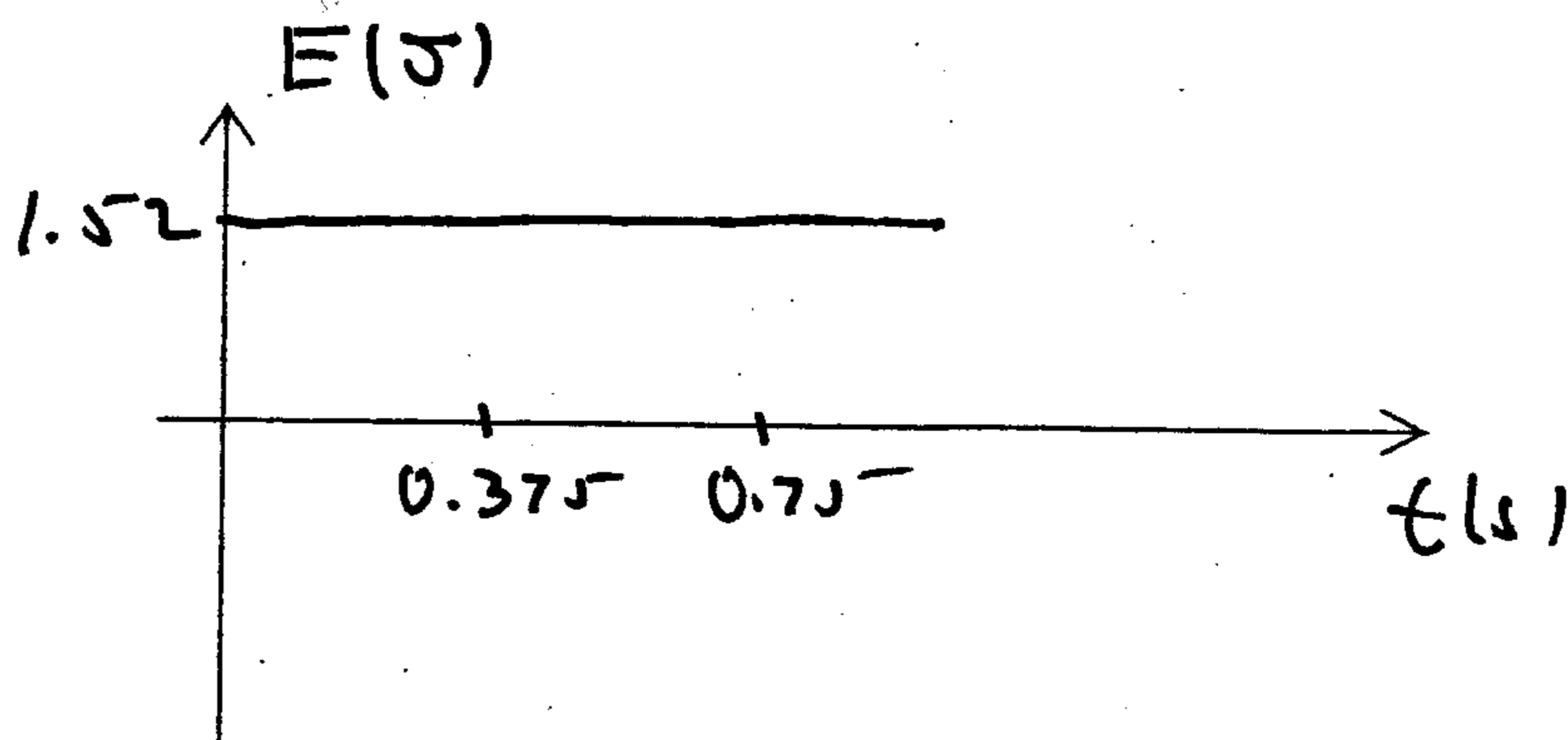
$$U(t) = U_m \sin^2 \omega t$$

$$U_m = \frac{1}{2} k X_m^2 = \frac{1}{2} m \omega^2 X_m^2$$

$$U_m = \left(\frac{1}{2}\right)(0.75 \text{ kg})(8.38 \text{ s}^{-1})^2(0.24 \text{ m})^2 = 1.52 \text{ J}$$

4

d) In the space below, sketch a graph of total mechanical energy versus time.



e) From the original graph, the maximum speed is seen to be 2.0 m/s. Suppose this speed is increased to 4.0 m/s. What is the period of oscillation of the oscillator?

$$T = 0.75 \text{ s independent of } V_m, X_m, \text{ etc.}$$